Congestion Pricing

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Motivation

- Roadway congestion causes emissions, lost productivity, fuel wastage, increased logistics costs, etc.
- A part of this congestion is caused by inefficient choices of individual drivers.
- We observed that there is more efficient way of routing drivers (using System Optimal assignment) in the network as compared to User Equilibrium assignment.
- We also observed that one can charge toll equal to $\gamma x_{ij}^{SO} \frac{dt_{ij}(x)}{dx}\Big|_{x_{ij}^{SO}}$, where γ is VOT in ₹/min to achieve SO state. Such pricing mechanism is known as First-best pricing which state that drivers should pay a toll equal to difference between marginal social cost and marginal private cost.
- This will help avoiding the negative externalities caused by any driver on other other drivers in the network.
- The initial idea of road pricing was put forward by Pigou (1920).
- Congestion pricing can help generate revenue for other Government initiatives.

Second-best pricing

- In First-based pricing mechanism, as a toll designer, you are allowed to charge toll on possibly every link. Is this practical?
- We could also try selecting a subset of links in the network to affect the route choice of drivers and bring the assignment closer to SO state (if not SO state). Such pricing mechanism is also known as Second-best pricing. The key questions are:
    1. Where to charge toll?
    2. How much toll to charge?
    3. How will it impact the travelers?

Example: Singapore

Singapore was the first country to introduce congestion pricing (called Area Licensing Scheme) on its roads in 1975, and it was later refined in 1998 (called Electronic Road Pricing). It charges a toll (up to $5 during peak hours) for entering restricted zone.

Figure: ERP gantry (Source: Wiki)

Example: London

London Congestion Charge is a £15 daily charge if you drive within the Congestion Charge zone 7:00-18:00 Mon-Fri and 12:00-18:00 Sat-Sun and bank holidays. No charge between Christmas Day and New Year's Day bank holiday (inclusive)[1].

Figure: London congestion charge and congestion zone (Sources: oecd.org and onlondon.co.uk)

[1] Source: https://tfl.gov.uk/modes/driving/congestion-charge

Example: Stockholm

Stockholm congestion tax was introduced in 2006 which charges tax in the entire Stockholm City Centre, which includes Södermalm, Norrmalm, Östermalm, Vasastaden, Kungsholmen, Stora Essingen, Lilla Essingen and Djurgården.[2]

Time of day	Tax Inner City[7]	Tax Essingeleden [7]	
00:00 – 06:29	0 SEK	0 SEK	0 €
06:30 – 06:59	15 SEK	15 SEK	1.62 €
07:00 – 07:29	25 SEK	22 SEK	2.38 €
07:30 – 08:29	35 SEK	30 SEK	3.24 €
08:30 – 08:59	25 SEK	22 SEK	2.38 €
09:00 – 09:29	15 SEK	15 SEK	1.62 €
09:30 – 14:59	11 SEK	11 SEK	1.19 €
15:00 – 15:29	15 SEK	15 SEK	1.62 €
15:30 – 15:59	25 SEK	22 SEK	2.38 €
16:00 – 17:29	35 SEK	30 SEK	3.24 €
17:30 – 17:59	25 SEK	22 SEK	2.38 €
18:00 – 18:29	15 SEK	15 SEK	1.62 €
18:30 – 23:59	0 SEK	0 SEK	0 €

Figure: Gantry, congestion zone, tax rate (Sources: Wiki and itf-oecd.org)

[2]Source: Wiki

Example: New York

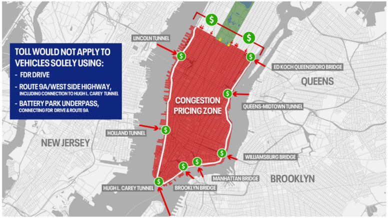
Central Business District Tolling Program was introduced in 2025. Since January 5, 2025, vehicles entering the Congestion Relief Zone in Manhattan—local streets and avenues at or below 60 Street—are being charged a toll[3].

When does NYC congestion pricing start?

Congestion pricing is scheduled to begin on Sunday, Jan. 5, 2025.

Congestion pricing map

Vehicles entering the Manhattan zone, which is local streets and avenues at or below 60 Street – near Central Park – will be charged a toll.



This map shows the proposed zone for New York City congestion pricing.

Drivers on the FDR and West Side Highway passing through the borough will not be charged.

How much will the toll cost?

Most vehicles would pay $9 to enter the congestion zone. It would come on top of the often-hefty tolls drivers pay to enter Manhattan via some bridges and tunnels.

Figure: Source: fox5ny.com

[3] www.mta.info

Congestion pricing strategies

- Cordon Pricing: Drivers are charged a fee each time they enter (sometimes exit) a defined boundary surrounding a congested area (typically CBD).
- Zonal Pricing: Drivers are charged based on the specific zone they travel within or between, often with different prices for different zones.
- Facility Pricing: Tolling on specific congested roads, bridges, or tunnels rather than an entire area. For example, HOV (High-Occupancy Vehicle) lanes in the U.S.
- Distance-based Pricing: Drivers are charged based on distance traveled.
- Dynamic Pricing: Toll rates vary dynamically by real-time congestion levels or by preset peak/off-peak periods.

Congestion pricing objectives

Some of the objectives are:

- Minimize the revenue
- If facilities are privately owned then maximize the revenue.
- Minimize the maximum toll charged on any facility
- Minimize the number of toll collection plazas (or stations)
- Maximize the equity in toll collection

Minimal-revenue congestion pricing

We observed that if we charge $c_{ij}(x_{ij}^{SO}) = \gamma x_{ij}^{SO} \frac{dt_{ij}(x)}{dx}\Big|_{x_{ij}^{SO}}$, where γ VOT (we assume same for all travelers) then UE and SO states coincides, i.e.,

$$\boxed{\underbrace{\left[\gamma \mathbf{t}(\mathbf{x}^*) + \mathbf{c}(\mathbf{x}^*)\right]^T (\mathbf{x} - \mathbf{x}^*) \geq 0, \forall \mathbf{x} \in X}_{UE} \implies \underbrace{\gamma \mathbf{t}(\mathbf{x}^*) x^* = \min_{\mathbf{x} \in X} \gamma \mathbf{t}(\mathbf{x})^T \mathbf{x}}_{SO}}$$

The tolls $c_{ij}(\mathbf{x})$ are known as marginal cost congestion prices. Although these tolls help achieve our objective to shift travelers from UE to SO but they do so by putting a high out-of-cost to the travelers. We can achieve the same objective by putting cheaper tolls since SO tolls are not unique. Given a choice, we want to select SO tolls $\mathbf{c}^*$ that collects the least revenue. Such tolls are known as minimum revenue tolls.

Algorithm for finding MR tolls for single-origin multiple-destination case (Dial (1993))

1. Solve the SO traffic assignment and build a subgraph of links with non-zero flow.
2. Find the topological order of these nodes and build a longest path tree by using the travel time (scaled by VOT) found in the SO traffic assignment solution. This step will produce longest path labels $D(i)$ (let's call these potential of nodes) for each node i.
3. Build a bush of links with positive potential difference. This can also potentially add links with zero flow.
4. Charge a toll equal to $c^*_{ij} = D(j) - D(i) - \gamma t_{ij}(x^{SO}_{ij})$.

LP formulation for finding MR tolls for single-origin multiple-destination case (Dial (1993))

$$\min_{\mathbf{D},\mathbf{c}} \quad \sum_{(i,j)\in A} c_{ij} x_{ij}^{SO} \tag{1a}$$

$$\text{s.t.} \quad c_{ij} + \gamma t_{ij}(x_{ij}^{SO}) + D(i) - D(j) = 0, \forall (i,j) \in A : x_{ij}^{SO} > 0 \tag{1b}$$

$$c_{ij} + \gamma t_{ij}(x_{ij}^{SO}) + D(i) - D(j) \geq 0, \forall (i,j) \in A : x_{ij}^{SO} = 0 \tag{1c}$$

$$c_{ij} \geq 0, \forall (i,j) \in A \tag{1d}$$

Finding MR tolls for multiple-origin multiple-destination case (Dial (2000))

- For multiple-origin case multiple-destination case, we can extend the previous LP. Let $D(i)^k$ be the potential of node i for origin k. Let s_{ij}^k be the surplus variable and let $z_{ij}^k = 0$ if $(x_{ij}^{SO})^k > 0$, otherwise 1 be the zero flow indicator variable.
- The constraints (1b)-(1c) can be replaced by

$$c_{ij} + \gamma t_{ij}(x_{ij}^{SO}) + D^k(i) - D^k(j) - z_{ij}^k s_{ij}^k = 0, \forall (i,j) \in A$$

- The dual of this LP is a max-flow problem which can be efficiently solved.
- Read the following paper for more details:
Dial, Robert B. "Minimal-revenue congestion pricing part II: An efficient algorithm for the general case." Transportation Research Part B: Methodological 34.8 (2000): 645-665.

Second-best congestion pricing

- In this pricing mechanism, we are allowed to toll only a subset of links A_T, e.g., links crossing the zonal boundary.
- These problems are generally modeled as Bi-Level Programming Problems (BLPP) or Mathematical Programs with Equilibrium Constraints (MPEC). The upper-level problem is the toll design problem and the lower-level problem is the traffic assignment problem that takes the tolls into account.

$$\min_{\mathbf{c}} \quad \sum_{(i,j)\in A} t_{ij}(x_{ij})x_{ij} \tag{2a}$$

$$\text{s.t.} \quad c_{ij} = 0, \forall (i,j) \in A \backslash A_T \tag{2b}$$

$$\mathbf{x} \in \operatorname{argmin}_{\mathbf{x}\in X} \left\{ \sum_{(i,j)\in A} \int_0^{x_{ij}} (\gamma t_{ij}(x) + c_{ij}) dx \right\} \tag{2c}$$

(2d)

Second-best congestion pricing

$$\min_{\mathbf{c}} \quad \sum_{(i,j)\in A} t_{ij}(x_{ij})x_{ij} \tag{3}$$

$$\text{s.t.} \quad c_{ij} = 0, \forall (i,j) \in A \backslash A_T \tag{4}$$

$$\mathbf{x} \in \operatorname{argmin}_{\mathbf{x}\in X} \left\{ \sum_{(i,j)\in A} \int_0^{x_{ij}} (t_{ij}(x) + c_{ij})dx \right\} \tag{5}$$

- We can develop a sensitivity-based heuristic to solve the above problem.
- Note that $\mathbf{c}$ is assumed to be transfer payments. The total sum of payments among players is zero.
- UE in the lower-level problem can be replaced by optimality conditions or VI problem.
- Finally, one can also include binary decision variables $y_{ij} = \begin{cases} 1, & \textbf{if toll is charged} \\ 0, & \textbf{if toll is not charged} \end{cases}$ and include constraints that make the shape of the zones (or corridors). This way optimization will also determine the best links to charge tolls.

Value of time and congestion pricing

- Value of time plays a crucial role in congestion pricing as it captures how drivers make tradeoffs between time and money.
- We assumed same VOT for each driver. However, different drivers have different VOT. It can be a consequence of socio-economic conditions of drivers.
- For heterogeneous divers with varying VOTs, we can assume a probability distribution of VOT (proportion of passengers with different values of VOT).
- Dial (1996) proposed a bi-criterion traffic assignment procedure in which instead of performing all or nothing assignment, we assign passengers on bi-criterion shortest paths (set of paths that minimizes generalized cost for travelers with different VOT).
- We can use that model for coming up with the toll values. I suggest reading the papers on the next slide.

Suggested reading

- Small, Kenneth A., Erik T. Verhoef, and Robin Lindsey. The economics of urban transportation. Routledge, 2024. Chapter 4
- Verhoef, Erik T. Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points. Transportation Research Part B: Methodological 36.8 (2002): 707-729.
- Dial, Robert B. "Network-optimized road pricing: Part I: A parable and a model." Operations Research 47.1 (1999): 54-64.
- Dial, Robert B. "Minimal-revenue congestion pricing part II: An efficient algorithm for the general case." Transportation Research Part B: Methodological 34.8 (2000): 645-665.
- Dial, Robert B. "Minimal-revenue congestion pricing part I: A fast algorithm for the single-origin case." Transportation Research Part B: Methodological 33.3 (1999): 189-202.

- Dial, Robert B. "Minimal-revenue congestion pricing part II: An efficient algorithm for the general case." Transportation Research Part B: Methodological 34.8 (2000): 645-665.

Thank you!