Pricing and revenue management

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Outline

Introduction

Price optimization

Price differentiation

Constrained supply

Revenue management

Introduction

- Pricing can help increase the supply chain profits by better matching the supply and demand.
- Revenue management is the use of pricing to increase the supply chain surplus and profit generated from limited supply chain assets. It can be applied to products/service with one or more following properties:
  - Customer segmentation (value of product varies in different market segments)
  - Perishable product
  - There is fixed capacity

  Example(s). Airlines and hotel industry

Demand function

It tells us how demand varies with the price of the product. We make the following assumptions.

- Demand function is defined for non-negative price.
- Demand function is downward slopping, i.e., if price goes up, the demand goes down. (Not true for all products!)
- Demand function is continuous function of price and is bounded by minimum and maximum demand.
- Demand function is differentiable (too make our life easier).

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Notations

- p: price of the product
- c: cost of one unit product to the seller
- $d(p)$: demand as a function of price
- $\Pi(p)$: profits to the seller for selling the product at price p

Basic price optimization

$$\max_p \ \Pi(p) = \underbrace{d(p)}_{\text{demand at price } p} \times \underbrace{(p-c)}_{\text{marginal profit}}$$

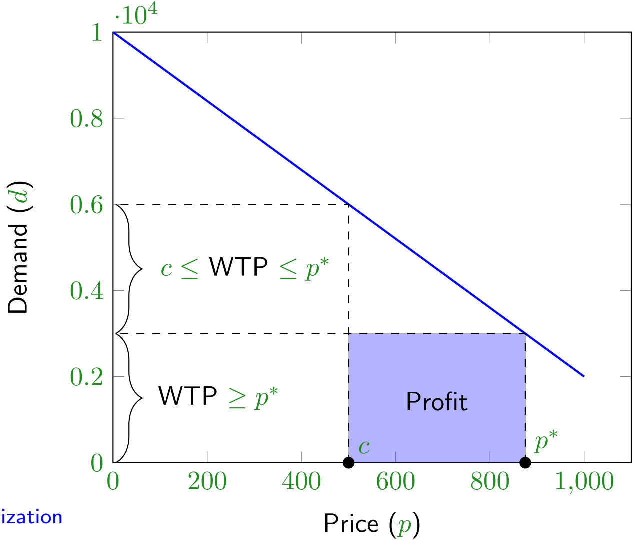
To maximze profits, we equate the derivative wrt p equal to zero.

$$\begin{aligned} \Pi^{'}(p) &= 0 \\ d^{'}(p^*) \times (p^* - c) &= 0 \\ \underbrace{p^* \times d^{'}(p^*) + d(p^*)}_{\text{marginal revenue}} &= \underbrace{c \times d^{'}(p^*)}_{\text{marginal cost}} \end{aligned}$$

At optimal price p^*, the marginal revenue is equal to the marginal cost.

Basic price optimization

Definition (Willingness to pay (WTP)). WTP is the maximum price at which a customer will buy one unit of product.



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Price differentiation

Definition (Price differentiation). practice of charging different prices to different customers for exactly same or slightly different version of a good/service.

Remark. It can be dangerous if not managed correctly due to arbitrage, cannibalization, and customers' reaction of being unfair.
Example(s). Airlines charges more prices for a given flight as its departure approaches.

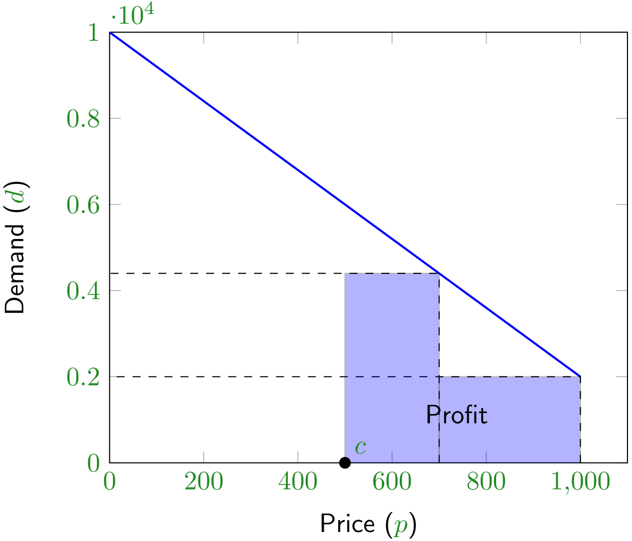
Price discrimination is different from dynamic pricing which means varying prices over time.

- Not always true. e.g., ridesharing charging different prices in different regions during peak hours is dynamic pricing.

Remark. Art lies in finding segmentation of customers so that seller can charge higher prices to high-WTP customers and low prices to low-WTP customers.

Price differentiation

If we charge ₹700 to lower WTP and ₹1000 to higher WTP



Price differentiation optimization

$$\boxed{\max_{p_1, p_2} \Pi(p_1, p_2) = (p_1 - c) \times d(p_1) + (p_2 - p_1) \times d(p_2)} \qquad (1)$$

Now, let's assume that there is a natural segmentation of customers. One group has high WTP who is willing to pay more than v (exogenous) and another group has low WTP who is willing to pay less than v. We can decompose the total demand function $d(p) = D - ap$ as:

1. $d(p_1) = D - ap_1 - d(v)$
2. $d(p_2) = \min\{d(v), D - ap_2\}$

Then, solve the basic price optimization model to obtain the optimal price.

Segmentation

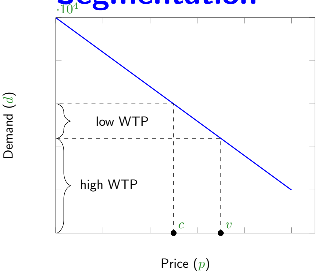
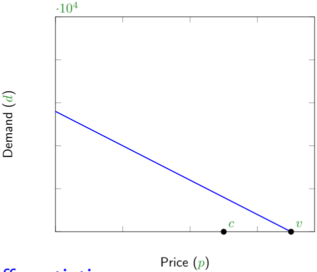
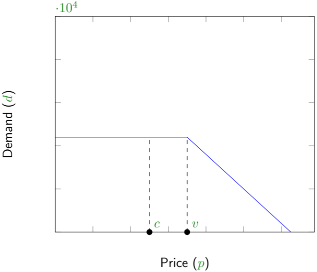


Figure: $d(p) = D - ap$



(a) $d(p_1) = D - ap_1 - d(v)$



(b) $d(p_2) = \min\{d(v), D - ap_2\}$

Limits to price differentiation

- Imperfect segmentation: It is difficult to predict precise WTP and so is market segmentation.
- Cannibalization: High WTP customers finding a way to buy at price set for low WTP.
- Arbitrage: Arbitrageurs can buy the product at low WTP and resell to high WTP customers below the market price and keeping the differential.

Strategies for price differentiation

- Group pricing (e.g., student discount, senior citizen discount, family specials, etc.)
- Channel pricing (e.g., internet versus physical stores)
- Regional pricing (e.g., South Delhi versus Old Delhi)
- Couponing and self selection (e.g., coupons for movies, sales for clothes)
- Product versioning (e.g., Amazon prime lite vs Amazon prime)
- Time-based differentiation (e.g., One-day v/s One-week delivery, Airline tickets closer to departure date)

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Pricing with constrained supply

- When there are limits on supply, seller may not satisfy all customers.
- Which customers should the seller be serving?
  - Do nothing and give product on FCFS basis
  - Allocate limited supply to favored customers using some mechanism such as lottery.
  - Raise the price until the demand falls to meet the supply. In other words, allocate supply to high WTP customers at high price.

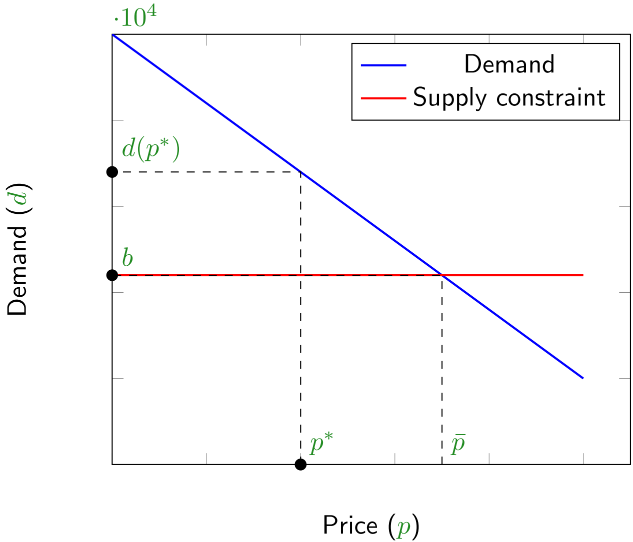
$$\begin{aligned} \max_{p} \quad & d(p) \times (p - c) \\ \text{subject to} \quad & d(p) \leq b \end{aligned} \tag{2}$$

where, b is the maximum supply.

- One can approach solving the problem (2) as follows. Solve (2) without the capacity constraint to get p^*. If $d(p^*) \leq b$, declare p^* as the optimal price. If $d(p^*) > b$, then find the runout price $\bar{p}$ by solving $d(\bar{p}) = b$ and declare $p^* = \bar{p}$.

Remark. The reduction in profit due to a supply constraint is called the opportunity cost associated to that constraint.

Price with constrained supply



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Revenue management

Definition (Revenue management). Strategy and tactics used by various industries (airline, hotel, etc.) to manage the allocation of their capacity to different fare classes over time in order to maximize revenue. Revenue management is applicable when all of the following conditions hold.

- Seller is selling a fixed stock of perishable capacity
- Customers book capacity prior to a deadline
- Seller manages a set of fare (booking) classes, each of which has fixed price (at least for a short duration)
- Seller can change the availability of fare classes over time

Remark. Revenue management is based on not setting and updating the prices but setting the availability of fare classes (price for each class remains same constant throughout the booking period).

Example: Air India

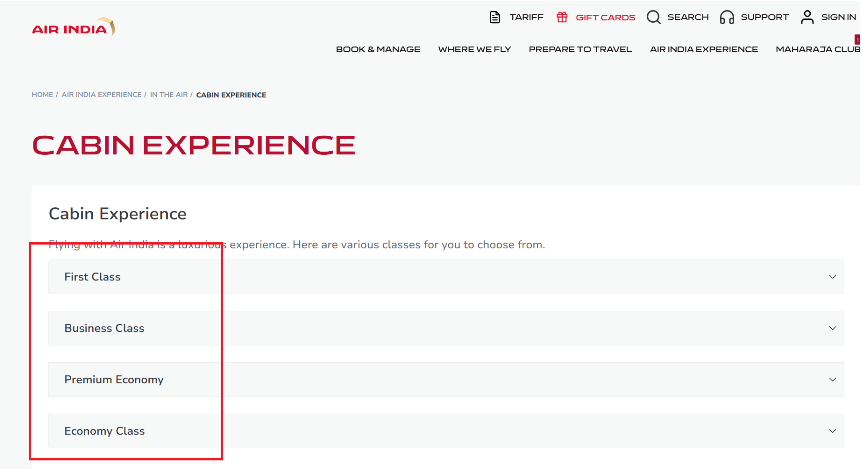


Figure: Wesbite: https://www.airindia.com/in/en/experience/in-air/amenities.html accessed on Feb 10, 2024 12:21

Example: Delta

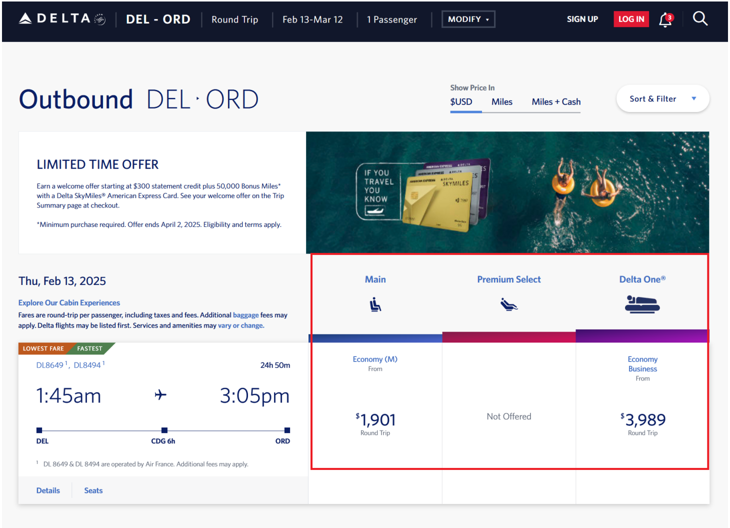


Figure: Wesbite: https://www.delta.com/flightsearch/search-results?cacheKeySuffix=5a1aa79a-e1ec-4b89-98de-ba66c469b96a accessed on Feb 10, 2024 12:25

Example: Marriott San Francisco

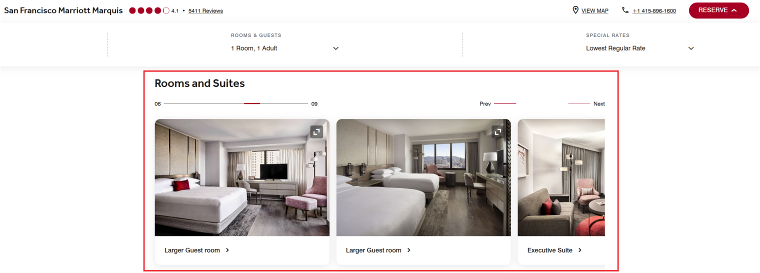


Figure: Wesbite: https://www.marriott.com/en-us/hotels/sfodt-san-francisco-marriott-marquis/overview/ accessed on Feb 10, 2024 12:34

Example: IRCTC

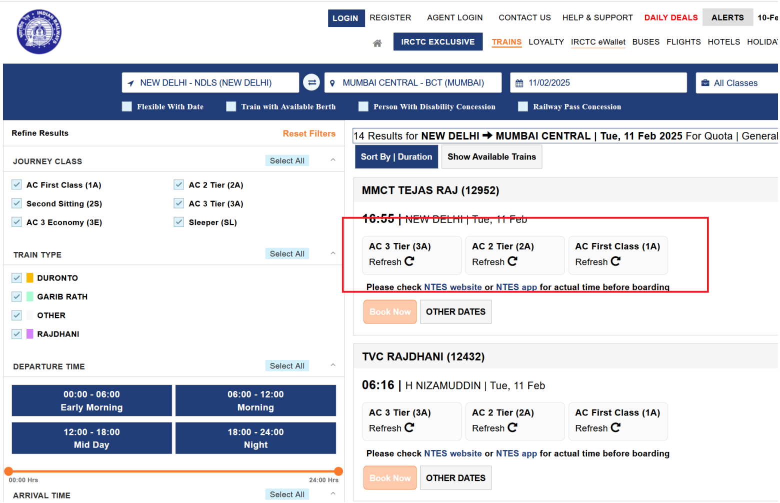


Figure: Wesbite: https://www.irctc.co.in/nget/booking/train-list accessed on Feb 10, 2024 12:40

Capacity allocation

Definition (Capacity allocation). Problem of determining how many seats (or hotel rooms) to allow low-fare customers to book if there is a possibility of future high-fare demand.

Two-class problem

- A firm with fixed capacity C serves two classes of customers, namely
  1. Discount customers (who generally book early)
  2. Full fare customers (who generally book later)
- Each discount customer pays $p_d > 0$ and full fare customer pays higher fare $p_h > p_d$
- $F_d(\cdot)$ and $F_h(.)$ are CDFs of discount customers and full fare customer demand.
- Assume that all discount booking requests occur before any full fare requests (otherwise problem is trivial!)

The problem is to determine how many discount customers (if any) we should allow to book so as to maximize the revenue., i.e., we need to determine booking limit b^* (maximum no. of discount booking) and the protection level $y^* = C - b^*$.

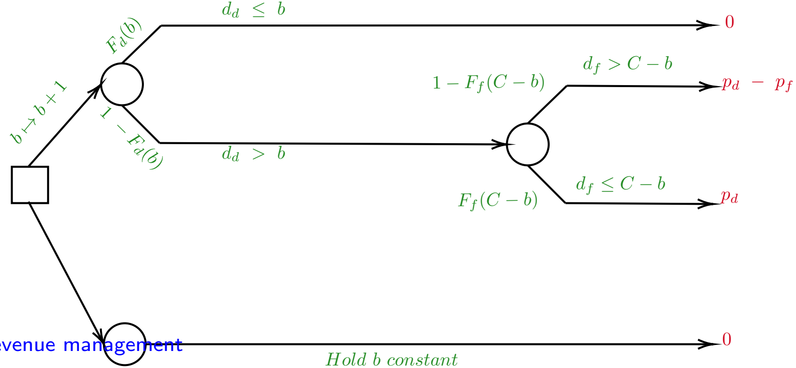
Remark. If b^* is too low, then we may turn away discount customers and fly our plane with empty seats (also referred to as spoilage). If b^* is too high, then we may turn away higher revenue customers (also referred to as dilution).

Decision tree approach

Let us assume that we set the booking limit as $0 \leq b < C$, the question is whether to increase the booking limit by 1, i.e. $b \mapsto b+1$? This depends on the expected change in revenue, which is evaluated below:

$$\mathbb{E}\left[h(b)\right] = F_d(b) \times 0 + \bar{F}_d(b)\left[\bar{F}_f(C-b)(p_d - p_f) + F_f(C-b)p_d\right] \quad (3)$$

$$= \bar{F}_d(b)\left[p_d - \bar{F}_f(C-b)p_f\right] \quad (4)$$



Simple hill-climbing algorithm

Remark. As long as the term $p_d - \bar{F}_f(C-b)p_f > 0$, we should keep increasing b by 1 until $b = C$ or $p_d - \bar{F}_f(C-b)p_f \le 0$. This suggests the following hill-climbing algorithm:

Input: $p_d, p_f, F_d(\cdot), F_f(\cdot), C$
Output: b^*
$b \leftarrow 0$
while $b < C$ **do**
    if $p_d - \bar{F}_f(C-b)p_f \le 0$ or $F_d(b+1) == 1$ **then**
        $b^* \leftarrow b$
        break
    else
        $b \leftarrow b+1$
    end if
end while
return $b^*, y^* = C - b^*$

Littlewood's rule

Above algorithm finds the largest value of b^* such that $\bar{F}_f(C-b) > \frac{p_d}{p_f}$. This can be approximated with following Littlewoods's rule.

$$\boxed{y^* = \min\left\{F_f^{-1}\left(1 - \frac{p_d}{p_f}\right), C\right\}} \tag{5}$$

Remark. This is also related to Newsvendor model by selecting $c_o = p_d$ and $c_s = p_f - p_d$.

$$F(y) = \frac{c_s}{c_s + c_o} = \frac{p_f - p_d}{p_f} = \left(1 - \frac{p_d}{p_f}\right)$$

Other realistic and complicated models

- Multiple fare classes
- No show and cancelation
- Overbooking
- Network management
- Strategic customers

Credits

- Pricing and revenue optimization by Robert l. Phillips, Second Edition
- Chopra et al. chapter on smart pricing
- IE5551 lecture taught by Xiaotang Yang

Thank you!